INTEGRATION OF A MULTIZONE AIRFLOW MODEL INTO A THERMAL SIMULATION PROGRAM

Rasmus L. Jensen 1, Karl Grau 2, and Per K. Heiselberg 1

1 Department of Civil Engineering, Aalborg University, Denmark
2 Danish Building Research Institute, Aalborg University, Denmark

ABSTRACT

An existing computer model for dynamic hygrothermal analysis of buildings has been extended with a multizone airflow model based on loop equations to account for the coupled thermal and airflow in natural and hybrid ventilated buildings.

In water distribution network and related fields loop equations have been widely used to resolve the flow of water and other fluids. In the field of natural ventilation the loop equation method have rarely been used in spite of its quality. Mainly due to problems with automatically setting up the independent loops.

The model described in this paper uses a methodology adopted from water distribution network that automatically sets up the independent loops and is easy to implement into a computer program.

Finally an example of verification of the model is given which demonstrates the ability of the models to accurately predict the airflow of a simple multizone building.

KEYWORDS
Spanning tree, loop equations, natural ventilation, hybrid ventilation, BSim.

INTRODUCTION

In multizone airflow simulation the common way to calculate the airflow is to set up a system of equations linking the pressure at each zone through airflow components (e.g. windows doors and cracks) and solve for the internal pressure (pressure node method). In water distribution network and related fields loop equations have been widely used to resolve the flow of water and other fluids, e.g. (Jeppson 1976).

When the airflow is solved by use of loop equations all the driving pressures (wind and stack) and pressure losses for the flow components are summed around the loop starting at an arbitrary node and following the loop back to the same node. Since returning to the same node the sum of all the pressure changes must be zero. The mass flow is then determined so this is fulfilled.

The theory of establishing the loops is based on graph theory. The goal is to set up a number of independent loops. This is done by first of all making a spanning tree that includes as many arches as possible without forming any loops. Each of the remaining arches then forms an independent loop together with a part of the spanning tree. The number of independent loops is equal to the number of flow components minus the number of zones plus one (Barlow et al 1969).

The method used to form the independent loops consist of “Making a spanning tree” by using a “Depth-first-search” and then “Creating the independent loops” by using a “Breath-first-search”. The loops are automatically generated by the program without any user involvement.

An idealised control is implemented which controls the opening degree of the windows and doors (window doors). The window doors have a finite number of opening positions, which can bee specified by the user. At each time step the control is allowed to change the opening positions of the window doors a finite number of times in an attempt to obtain the desired airflow needed to match the set-points of indoor air temperature and CO2 level. The control is not discussed in this paper, but it documented in Jensen (2005).

BSIM

The new model is integrated into an ISO STEP based, integrated building design tool, BSim (Wittchen et al. 2005).

The core of the design tool is a common building data model shared by the different design tools, and a common database with typical building materials, constructions, windows and doors. Figure 1 illustrates the user interface of BSim.
Applications integrated in BSim are:

- **SimView** for editing and visualisation of the building model geometry.
- **XSun** for analyses of solar distribution and shadows in and around buildings.
- **tsbi5** for thermal simulations. The program is based on the finite control volume method. It has been widely validated and employed in several international research projects, e.g. EU COMBINE (Augenbroe 1995). The program includes a model for synchronous calculation of transient hygrothermal conditions (Rode and Grau, 2003).
- **SimLight** a simple tool for estimation of the daylight conditions in convex spaces.
- **SimPv** a tool for calculation of the potential electrical production from building integrated PV-systems (Wittchen 2003).
- **SimDB** is the user interface for the common database with building materials, constructions etc.

The building model can be exported to the **Radiance** program for visualisation and detailed calculation of the lighting and daylighting conditions.

The multizone airflow model is fully integrated into the graphical user interface of BSim and requires only a few extra user data compared to the thermal model itself. At this time the airflow model includes only the orifice flow model.

**Data Model**

In BSim a building is represented as a collection of one or more rooms. A room is defined by the plane faces that separate the rooms from each other and from the outdoors. The faces define the geometry, and represent the location of constructions that are defined by their thermophysical properties of their materials. A face may include a number of windows, doors and openings. One or more rooms can be included in a so-called thermal zone (or just "zone"), indicating that the rooms take part in a thermal simulation. To a zone can be attached different systems that have an influence upon or control the indoor climate. A "system" is a general concept, and not just a mechanical system like ventilation, lighting or heating and cooling systems, but may also represent the heat and moisture load from people, or air exchange with the outside or neighbouring zones. In figure 2 the data model for a building is shown as a NIAM diagram (Nijssen and Halpin 1989).

The common data model for BSim consists of eight separate, but interrelated models of different aspects, e.g. a model for 3D geometry and topology. The models are defined as EXPRESS files, which have been automatically converted to C++ classes, on which the implementation of the program is based (Rode and Grau 1996).

**LOOP EQUATIONS IN BUILDING VENTILATION**

Few people have worked with loop equations within the field of natural ventilation special in the last few decades. The most frequently used argument not to use the loop equation method is that forming of the independent loops is a difficult task, e.g. (Axley 2001) and difficult to automate in computer programs.

Axley (2001) reintroduced the loop equation method to the field of natural ventilation focusing on design rather than simulation of natural ventilation systems. The reader is referred here for a thorough introduction to the loop equation method.

Comparing the loop equation and pressure node method both the mathematical formulation and the number of equations needed to be solved is different. In connection with the development of AIRNET
Walton (1989) investigated and compared the two methods including implementation of different types of airflow components. For some elements it was easy to change from one method to the other, but for others one or the other method gave a more straightforward formulation. Both methods have advantages and both methods should be considered.

Wray and Yuill (1993) compared the number of equations for 50 different buildings. For a few buildings the number of equations that needed to be solved differed up to approximately 50. For most buildings the differences was much smaller (up to 10) and in average the number of equations for the two methods was similar.

As mentioned above there are differences between using the pressure node and the loop equations. However since there are no commercial programs using the loop equations methods the experience towards which of the methods that are superior if any is limited. The choice of using the loop equations in this work was done to be able to explore the difference between the two methods.

FORMING THE LOOPS

Forming the loops consist of two parts. The first part is forming a so called Spanning Tree that includes all the nodes (different thermal zones) and as few arcs (openings) as possible. The second part is forming the independent loops. Each of the arcs not used for the spanning tree will together with part of the spanning tree form an independent loop.

Therefore the task is to setup a spanning tree and then use the remaining arcs to form the simplest (shortest) independent loops together with part of the spanning tree will form an independent loop.

FORMING THE SPANNING TREE

The spanning tree is formed by a "depth-first-search" where the starting point is the outdoor node. Then a random node connected to the outdoor node is chosen moving from node to node until a node is reached that is only connected to nodes in the spanning tree. The one follows the spanning tree back until a node is reach that is connected to a node not in the spanning tree. Then one continues creating the spanning tree from that node. This is done until all nodes are in the spanning tree. The structure of forming the Spanning Tree is listed below.

1. Associate label DFI(n) = 0, n = 1, 2, ..., N, with each node in the base graph
2. Set j = 1
3. Initialize a set of arcs within the growing tree \( A_s = [] \)
4. Identify the starting node \( n_s \) at random or as being the "root" of the base graph
5. Initialize a set of nodes contained within the growing tree \( C = \{n_s\} \), and set DFI\( (n_s) = j \)
6. Identify \( A_s(n_s) = \{\text{arcs in base graph connected to node } n_s\} \)
7. Identify end-points \( E(n_s) \) of the arch in \( A_s(n_s) \)
8. Eligible adjacent nodes to \( n_s \) are endpoints that are not contained within the growing tree; i.e., \( A(n_s) = E(n_s) - \{E(n_s) \cap C\} \)
9. If \( A(n_s) \) is an empty set, \( A(n_s) = {} \); i.e., there is no adjacent node that is not yet in the growing tree, then
   a. \( DFI(n_s) = 0 \)
   b. \( j = j - 1 \)
   c. Find a node \( n'_s \subset C \) for which \( DFI(n'_s) = j \)
   d. \( n_s = n'_s \)
   e. Go to step 6
10. If \( A(n_s) \) is a nonempty set, i.e. \( A(n_s) \neq {} \), then
   a. \( j = j + 1 \)
   b. Associate DFI\( (n) = j \) with each node \( n \subset A(n_s) \)
   c. Choose one node \( n'_s \) from \( A(n_s) \) at random
   d. \( C = C + \{n'_s\} \)
   e. \( A_s = A_s + \{\text{arc whose endpoints are } n_s \text{ and } n'_s\} \)
   f. \( n_s = n'_s \)
11. Repeat from step 6 until all nodes have been added to the connected set, i.e., a spanning tree is formed

FORMING THE INDEPENDENT LOOPS

The independent loops are formed by a "breath-first-search". The procedure is repeated for each arc not in the spanning tree. From one of the nodes of the arc all neighbouring nodes are connected. Then all the neighbours of the neighbouring nodes are connected until the second node of the arc is reached thereby forming the shorted loop. The structure of forming the independent loops is listed below.

1. Associate label BFI\( (n) = 0, n = 1, 2, ..., N, \) with each node in the base graph
2. Set \( j = 1 \)
3. Choose one of the arcs from the co-tree set \( a_i \in A_c \)
4. Initialize a set of arcs within the ring \( L = \{a_i\} \)
5. Identify the endpoints \( n_i \) and \( n'_i \) of the co-tree arc \( a_i \)
6. Initialize a set of nodes contained within a queue \( Q = \{n_i\} \), and set BFI\( (n) = j \)
7. Identify \( A_s = \{\text{arcs in the tree set } A_s \text{ connected to nodes in the queue } Q\} \)
8. Identify endpoints $E$ of the arcs in $A_n$ that are not labelled, i.e., $BFI(n) = 0$ for all $n \in E$

9. Set $Q = []$

10. 
   a. Set $j = j + 1$
   b. Associate $BFI(n) = j$ with each node $n \in E$
   c. Set $Q = E$

11. Repeat from step 7 until $n_i$' acquires a label

12. Retrace back from $n_i'$ to $n_i$ (using the tree arcs $A_j$) in decreasing order of $BFI$. The arcs belonging to the ring are stored in $L$

**MULTIZONE AIRFLOW MODEL**

In the following the theoretical basis for the multizone airflow model based on loop equations is described. The description accounts for the coupled thermal/airflow in natural and hybrid ventilated buildings.

**Conservation of mass**

For each node (thermal zone) conservation of mass must be meet. In this formulation the mechanical ventilations system acts as a source term.

$$\sum m_{in} - \sum m_{out} = m_{mech, removed} - m_{mech, supplied} \quad (1)$$

Where:

$m_{in}$ Mass flow into the zone through openings [kg/s]
$m_{out}$ Mass flow out of the zone through openings [kg/s]
$m_{mech, removed}$ Mass flow removed by mechanical ventilation [kg/s]
$m_{mech, supplied}$ Mass flow supplied by mechanical ventilation [kg/s]

**Conservation of energy**

For each loop the energy must be preserved, i.e. the loss of energy caused by friction (resistance) is equal to the energy supplied by wind and thermal buoyancy.

$$\Delta P_{loss} = \Delta P_{buoyancy} + \Delta P_{wind} = \sum c_i \quad (2)$$

Where:

$\Delta P_{loss}$ Pressure loss in the ring from the openings [Pa]
$\Delta P_{buoyancy}$ Driving force from thermics [Pa]
$\Delta P_{wind}$ Driving force from wind [Pa]
$c_i$ Resistance of opening $i$ [Pa]

The demand of conservation of mass and energy can be expressed in a linear equation system

$$[M][Q] = [\Delta P] \quad (3)$$

Where the matrix $M$ consist of two parts, one for conservation of energy, and the second for conservation of mass. The vector $Q$ represents the volume flows, and the vector $\Delta P$ represents the pressure difference and the mass flow supplied by the mechanical ventilation system.

**Equation system**

The following gives a description of how the equation system is set up for a simple building. The building (see figure 3) consist of two zones TZ1 and TZ2. Both zones have an opening facing the outdoor. The separating wall has two openings allowing air to flow between the zones.

![Figure 3 Simple building with nodes and branches](image)

The resistance for each opening in the loop is calculated by the following formula, and the sign reflects the direction of the airflow:

$$c_i = \frac{\rho \cdot V_i^2}{2 \cdot C_d \cdot A_i^2} \quad (4)$$

Where:

$c_i$ Resistance of opening $i$ [Pa]
$\rho$ Air density [kg/m³]
$V_i$ Air flow through the opening [m³/s]
$C_d$ Discharge coefficient [-]
$A_i$ Area of the opening [m²]

$$\rho = \frac{m}{V} = 0.0034837 \cdot \frac{P}{T} - 0.0013169 \cdot \frac{P_d}{T} \quad (5)$$
Where:
\[ \rho \] Density \([\text{kg/m}^3]\]
\[ m \] Mass \([\text{kg}]\]
\[ V \] Volume \([\text{m}^3]\]
\[ p \] Partial pressure \([\text{Pa}]\]
\[ p_d \] Vapor pressure \([\text{Pa}]\]
\[ T \] Air temperature \([\text{K}]\)

The density differences are converted to pressure differences assuming hydrostatic pressure distribution.

\[
\Delta P_{e(\bar{z}_{i,j})} = -\int_{z_i}^{z_j} g \rho_e(z) dz
\]  
(6)

Where:
\[ \Delta P \] Pressure difference \([\text{Pa}]\)
\[ g \] Acceleration due to gravity \([\text{m/s}^2]\)
\[ z \] Height \([\text{m}]\)

When the temperature is constant, the formula can be expressed as:

\[
\Delta P_{e(\bar{z}_{i,j})} = -\rho_e g \Delta z_{i,j}
\]  
(7)

The driving pressure due to wind is calculated the normal way as

\[
P_{\text{wind}} = C_p \frac{\rho U_{\text{ref}}^2}{2}
\]  
(8)

Where:
\[ C_p \] is the pressure coefficient on the façade
\[ U_{\text{ref}} \] the reference wind velocity \([\text{m/s}]\)

**Solving the equation system**

The focus of this work was not to test or investigate different schemes for solving the equations system. The main focus was on ensuring an accurate solution every time.

Savić and Walters (1996) concluded in their investigation that the Linear Theory Method (LTM) provided the best overall result. This method has therefore been implemented as a first method of solving the equations.

**INTEGRATION OF THE AIRFLOW MODEL INTO THE THERMAL MODEL**

In the literature different ways of integrating airflow and thermal models have been described. These can be divided into four categories (see eg. (Hensen 1995) and (Orme 1999))

- Sequential coupling
- Ping-pong
- Onions
- Full integration

In the first to methods there is no feedback within each individual time step and therefore these are not recommendable. Of the last two the Onion method was chosen in this case. The methods should give the same results therefore the choice depend more on the practical implementation.

In this case the airflow model was integrated into an existing simulation program and thermal model. The choice of the Onion approach caused minimum change to the thermal model, independent setup of the airflow model and the possibility to easily implement another airflow model if desired.

**Computation regime**

The use of the Onion method implies the use of iterations both in the individual models and between the models. The overall calculation procedure consists of three parts:

- Calculation of temperatures and desired airflows to fulfill the set-points of e.g. temperature or CO2 (thermal model)
- Calculation of possible airflow rates and necessary opening area of the windows (airflow model)
- Iteration between the models if the desired airflows can not be fulfilled, resulting in change of temperature in the thermal zones.

**VERIFICATION**

The implementation of the airflow model has been verified carefully. The verification has included verification of the generated spanning tree and loops and of the calculated airflows. At the moment the model is being tested in practice.

**Example of verification**

In this example the simulation period was one week with the Danish design reference year as weather data. Normally results from the program are only available on one hour interval but for this purpose results are shown for all the time steps, four in this case.

The verification included the following parameters

- Driving pressure from wind
- Driving pressure from thermal buoyancy
- Airflow through the openings
- Mass flow through the openings
- Resistance of the openings

The building model used was that shown on figure 3 and the results from the run were compared to hand
calculations using the weather data and calculated temperatures. On figure 4 the difference in mass flow both relative and absolute between the simulation results and the hand calculation is shown. The difference is due to rounding error both in the program and the calculation.

In figure 5 the airflow in thermal zone 1 (TZ1) is given. Negative values indicating that the air comes from TZ2. As can be seen from the figures both large and small airflows are handle well as is the change of direction of the air flow.

CONCLUSION

An existing whole building thermal simulation tool has been extended with a multizone model for calculation of the airflow between thermal zones in the building. This makes it possible to predict the airflow through openings between zones and through openings in the building enclosure.

The model is based on the loop equation method, which has been widely used to resolve the flow of water in water distribution networks, and related fields. In the field of ventilation the loop equations was reintroduced in 2001 by [Axley] for the purpose of designing natural ventilation systems. Through this work loop equations has now also been introduce in simulation programs and thereby in the analysis face of designing naturally ventilated buildings.

The paper gives a short introduction to the thermal simulation program, to the theory for the multizone model. The adopted procedure for setting up the independent loops are described. Finally an example of verification is given that shows that the program is capable of accurately predicting the airflow.

The first experience with using the loop equations appears promising. Solving the equation system requires only few iterations to convert, typically less than 3, and no problems with converging has been detected in this preliminary stage. In comparison to the pressure node methods computation time is prolonged due to forming the loops, however this is done very fast and the entire computation time is only slightly increased.

ACKNOWLEDGMENT

The authors would like to thank Professor James W. Axley Yale School of Architecture for fruitful discussions and inspiration on loop equations.

REFERENCES


**Figure 4** Difference in mass flow between the simulation results and the hand calculation

**Figure 5** Airflow in thermal zone 1 (TZ1) (negative values indicating that the air comes from TZ2)