A MODEL FOR ADVECTION HEAT AND MOISTURE FLOWS IMPLEMENTED IN A PROGRAM FOR WHOLE-BUILDING HYGROTHERMAL SIMULATION

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ABSTRACT
A model for calculating exfiltration & infiltration air flows in exterior building envelope constructions has been implemented in the whole-building hygrothermal simulation tool BSim. The tool is able to predict indoor humidity conditions using a transient model for the moisture conditions in the building envelope.

INTRODUCTION
When air flow passes through the building envelope, it has some notable influence on the heat flow through the wall. However, the impact on the moisture conditions can, even for small air flows, completely change the situation, e.g. from a healthy construction to a disaster. Air flows from warm/humid environment towards colder climates should generally be avoided, whereas air flows in the opposite direction could have a drying effect.

When infiltrating air flows can be seen to provide some of the required fresh air supply to the indoor environment, then the so-called dynamic wall principles applies, and the wall itself works as a heat exchanger where some of the heat lost by conduction is recovered by the ingoing air.

For these reasons there is a desire to implement a model for advective heat and moisture flow in a whole building simulation tool. BSim from the Danish Building Research Institute has been chosen as one such tool. BSim has been presented previously to IEA Annex 41 (Rode & Grau, 2003 and 2004) and in Grau & Rode, 2005. A multizone model for air flow between zones (rooms) within a building has been implemented into BSim as described previously by Grau & Rode, 2005. For more information about BSim, see www.bsim.dk.

MODEL FOR INFILTRATION & EXFILTRATION AIR FLOWS
The infiltration and exfiltration air flows in exterior constructions is dependent on the wind speed and wind direction, as well as the surroundings of the building.

Filtration through small cavities in a construction can be expressed according to the Classic Orifice method (Jensen, 2005) as:

\[ q_{inf} = k \cdot A \cdot \sqrt{2 \cdot \frac{\Delta P}{\rho}} \]

(1)

where

- \( q_{inf} \) Air volume, m³/s
- \( k \) Constant taking into account the friction of the cavity, -
- \( A \) Area of cavity, m²
- \( \Delta P \) Pressure difference, Pa
- \( \rho \) Air density, kg/m³

\[ \Delta P = p_n - p_i \]

(2)

where

- \( p_n \) Air pressure on the construction facing outside, Pa
- \( p_i \) Air pressure on inside, Pa

The air pressure can be expressed as

\[ p_n = \frac{1}{2} \cdot c_{p_n} \cdot \rho \cdot v^2 \]

(3)

where

- \( c_{p_n} \) Pressure coefficient on outside, -
- \( \rho \) Air density, kg/m³
- \( v \) Air velocity in reference height, m/s

and

\[ p_i = \frac{1}{2} \cdot c_{p_i} \cdot \rho \cdot v^2 \]

(4)

where

- \( c_{p_i} \) Pressure coefficient for the zone, -

Because of continuity, the sum of infiltration and exfiltration through constructions facing the zone must be zero, the pressure coefficient can be calculated.

\[ c_{p_i} = \frac{\sum_{k=1}^{n} A_k \cdot c_{p_k}}{\sum_{k=1}^{n} A_k} \]

(5)

where

- \( A_k \) Area of construction k facing outside, m²
Pressure coefficient for construction k, -
The filtration through a construction then can be expressed as
\[ q_{\text{inf,m}} = \frac{\text{fac} \cdot \sqrt{\frac{1}{2} \cdot |cp_n - cp_l|} \cdot v^2}{A_c} \]  \hspace{1cm} (6)

where
- \( q_{\text{inf,m}} \) Filtration, m\(^3\)/s/m\(^2\)
- \( \text{fac} \) A user given factor taking into account the area and friction of the cavities in the construction, -
- \( A_c \) Area of the construction, m\(^2\)
- \( cp_n - cp_l > 0 \) Infiltration
- \( cp_n - cp_l < 0 \) Exfiltration
The pressure coefficients are dependant of the orientation and slope of the construction, the wind direction, and of the surroundings, sheltered or exposed.
In BSim there is tables of typical pressure coefficients that will be used. The wind speed is dependent on the height of the building, and of the terrain type, e.g. opens flat country, or located in a city.

It is realized that there could be a need for implementation of another air flow model, such as the air flow is governed by Darcy flow through porous wall materials. This would give some other air flow rates, where the rate is proportional to the pressure difference across the building envelope, as opposed to in the here implemented model where it is proportional to the square root of the pressure difference.

PROCEDURES TO CALCULATE HEAT AND MOISTURE TRANSPORT THROUGH POROUS WALLS WITH A FILTRATING AIR FLOW

The filtration air flow \( q_{\text{inf,m}} \) is calculated according to the procedures described in the previous section.

Both temperature and vapour pressures are calculated in BSim using an implicit finite control volume scheme

For heat conduction, the finite difference form of the heat balance equation for one control volume looks:
\[ \rho c_p \frac{T_{i+1}^{j+1} - T_i^j}{\Delta t} = \frac{\Delta x}{2 \lambda} \left( \frac{T_{i+1}^{j+1} - T_{i-1}^{j+1}}{2\lambda_{i+1}} + \frac{T_{i+1}^{j+1} - T_{i-1}^{j+1}}{2\lambda_{i-1}} \right) + q_{\text{air}}(p_i^{\text{up}} - p_i^{j}) \] \hspace{1cm} (7)

where:
- \( \rho \) Density of the material \hspace{1cm} \text{kg/m}^3
- \( c_p \) Specific heat \hspace{1cm} \text{J/(kg K)}
- \( \Delta x \) Width of the control volume \hspace{1cm} \text{m}
- \( T \) Temperature \hspace{1cm} \text{K}
- \( \Delta t \) Time step \hspace{1cm} \text{s}
- \( \lambda \) Thermal conductivity \hspace{1cm} \text{W/(m K)}
- \( R \) Possible interface resistance \hspace{1cm} \text{m}^2 \cdot \text{K/W}
- \( i \) Index for control volume number
- \( j \) Index for time step

With air flow, equation (7) is modified as follows:
\[ \rho c_p \frac{T_{i+1}^{j+1} - T_i^j}{\Delta t} = \frac{\Delta x}{2 \lambda} \left( \frac{T_{i+1}^{j+1} - T_{i-1}^{j+1}}{2\lambda_{i+1}} + \frac{T_{i+1}^{j+1} - T_{i-1}^{j+1}}{2\lambda_{i-1}} \right) + q_{\text{air}}(p_i^{\text{up}} - p_i^{j}) \] \hspace{1cm} (8)

where:
- \( T_{\text{upstream}} \) Temperature in the adjacent control volume from which the air comes

For vapour diffusion, the finite difference form of the moisture balance equation looks:
\[ \rho \xi \frac{p_i^{j+1} - p_i^j}{\Delta t} \frac{p_i^{j+1} - p_i^j}{\Delta t} = \frac{\Delta x}{2 \delta_i} \left( \frac{p_i^{j+1} - p_i^j}{\Delta t} + \frac{p_i^{j+1} - p_i^j}{\Delta t} \right) \] \hspace{1cm} (9)

where:
- \( \xi \) Moisture capacity \hspace{1cm} \text{kg/kg}
  - (slope of sorption isotherm)
- \( p \) Vapour pressure \hspace{1cm} \text{Pa}
- \( p_s \) Saturation vapour pressure \hspace{1cm} \text{Pa}
- \( \delta \) Water vapour permeability \hspace{1cm} \text{kg/(m s Pa)}
- \( Z \) Possible interface water vapour resistance between control volumes

With air flow, equation (9) is modified as follows:
\[ \rho \xi \frac{p_i^{j+1} - p_i^j}{\Delta t} = \frac{\Delta x}{2 \delta_i} \left( \frac{p_i^{j+1} - p_i^j}{\Delta t} + \frac{p_i^{j+1} - p_i^j}{\Delta t} \right) \] \hspace{1cm} (10)

where:
- \( p_{\text{upstream}} \) Vapour pressure in the adjacent control volume from
which the air comes

\[ R_v = \text{Gas constant for water vapour } \frac{J}{(\text{kg} \cdot \text{K})} = 461.5 \]

\[ T = \text{Absolute temperature } \text{K} \]

A “new” vapour pressure distribution has been determined. However, since the whole problem is non-linear (because the moisture capacity is not a constant value), a special procedure must be followed to ensure that the mass balance will be correct. The vapour pressures just found must be regarded as preliminary indications of the new values. It could even be that some of the preliminary vapour pressures have values that exceed the values of \( p_s \) at the same location, or they may be negative. This is either neglected at first (!), or the time step is repeated with a smaller \( \Delta t \). The found vapour pressures are used in the moisture balance of each control volume as follows.

The vapour flux across the interface between control volumes, \( i \) and \( i+1 \), for the time step from \( j \) to \( j+1 \) is:

\[
g_{i+1/i}^{j+1/2} = \frac{p_i^{j+1} - p_{i+1}^{j+1}}{\frac{\Delta x_{i+1}}{2} \delta_{i+1} + \frac{\Delta x_i}{2} \delta_i + Z_{i+1/2}}
\]

(11)

where:

\[ g \quad \text{Water vapour flux } \frac{\text{kg}}{\text{m}^2\text{s}} \]

The equilibrium relative humidity is determined for each control volume by taking the calculated moisture contents and using the sorption curve for the relevant material (that is, an inverse expression for the sorption curve will be used).

**EXAMPLE 1 - VALIDATION**

An existing BSim model from one of the variations of Common Exercise 1 (the BESTEST building) has been taken as starting point for illustrating the new model with filtration air flow in the building envelope. The building consists of 150 mm solid aerated concrete walls, and the walls have no surface coating.

One of the exterior walls of the building is split up such that half of the wall has a filtration air flow going through it, while the other is calculated as being air tight. See Figure 1. The building is exposed to an outdoor climate which artificially was constant at 20ºC and 30% RH, while the indoor climate was assumed operated so it was constantly 30ºC and 50% RH. Outside, the wind was assumed blowing from west at 5 m/s, so by turning the building so the façade with filtration air flow either faces the wind, or is on the leeward side, then the heat and moisture flows in the building envelope could be investigated with the advection phenomena. With constant boundary conditions and constant air flows, the results can be compared with results of analytical calculations. The air flows were adjusted to always give a mass flow rate of 0.0005 kg/(m²s), corresponding to 1.57 m³/(m²h) – either infiltrating or exfiltrating.

The mesh had control volumes of thickness 1.7 mm closest to the indoor climate, increasing to 10 mm through most of the wall, so there were a total of 17 control volumes to represent the wall.
Figure 1 BESTEST building modelled in BSim. The wall that is split in two halves has one half calculated with the filtration model, whereas the other half is calculated as being air tight.

The analytical solution for the temperature is given by (Hagentoft, 2001):

\[ T(x) = T_{\text{out}} + (T_m - T_{\text{out}}) \left(1 - e^{-\frac{x}{\ell}}\right) \]  

(13)

where

- \( L \) Thickness of the wall, m
- \( \ell \) A quantity of dimension length that characterises the interaction between conductive and convective heat flows, such that:

\[ \ell = \frac{\lambda}{c_{\text{air}}M_{\text{air}}} \]  

(14)

where

- \( c_{\text{air}} \) Heat capacity of air = 1005 J/(kg·K)
- \( M_{\text{air}} \) Mass flow rate of air kg/(m²·s)

Figure 2 shows the comparison between the temperatures calculated numerically using BSim and the analytical solutions for exfiltration, no air flow, and infiltration, respectively.

The table below gives a comparison between the analytical and numerically calculated temperatures in the middle of the wall.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Analytical</th>
<th>Numerical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exfiltration</td>
<td>25.52</td>
<td>25.55</td>
</tr>
<tr>
<td>No air flow</td>
<td>25.00</td>
<td>25.04</td>
</tr>
<tr>
<td>Infiltration</td>
<td>24.48</td>
<td>24.50</td>
</tr>
</tbody>
</table>
The temperature determines the saturation vapour pressure. An approximation of the analytical vapour pressure is found as:

$$p(x) = p_{out} + (p_m - p_{out}) \frac{e^{1 \ell} - 1}{e^{1 \ell} - 1}$$  \hspace{1cm} (15)

where

$$\ell = \frac{\delta}{c_v M_{aw}}$$  \hspace{1cm} (16)

$\ell$ is a quantity of dimension length that characterises the interaction between conductive and convective water vapour flows, such that:

$c_v$ is not quite constant since $p$ varies with location in the construction, and therefore, the analytical solution is only approximate (with a few percents’ deviation).

Finally, RH can be determined as $p/p_s$. Figure 4 shows the comparison between the temperatures calculated numerically using BSim and the analytical solutions for exfiltration, no air flow, and infiltration, respectively.

The table below gives a comparison between the analytical and numerically calculated relative humidity in the middle of the wall.

<table>
<thead>
<tr>
<th></th>
<th>Analytical</th>
<th>Numerical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exfiltration</td>
<td>66.8</td>
<td>64.9</td>
</tr>
<tr>
<td>No air flow</td>
<td>44.7</td>
<td>44.7</td>
</tr>
<tr>
<td>Infiltration</td>
<td>22.5</td>
<td>23.4</td>
</tr>
</tbody>
</table>
It can be concluded that the advection model in BSim predicts the analytical temperature distribution with a good satisfaction, while also the relative humidity distribution is predicted with a fair amount of accuracy.

EXAMPLE 2 – IMPACT ON ANNUAL CONDITIONS

In this example, all walls of the building were calculated as if they allowed for air infiltration and exfiltration, but now the building was exposed to the outdoor climate of Denmark, as it is described in the Design Reference Year for Copenhagen. A full year simulation was carried out, and the results of heat and moisture flows have been compared to the results of a similar calculation without filtration air flow. The building now has a heating system with a set-point of 20°C, and a cooling system (26°C). The 129.6 m³ big room has for 8 hours per day a moisture load of 500 g/h in (and 50 g/h the rest of the day). The air change rate is constant at 0.5 h⁻¹. The building is simulated without windows.

Figure 5 shows the monthly values of air exfiltration/infiltration for all four wall orientations. It is seen that exfiltration dominates except in the west wall. In February and March, however it is opposite, then infiltration dominates in the east wall, while there is exfiltration in the west wall. This corresponds with the average wind directions that can be seen in Figure 6.
Figure 5  Monthly average values over the year of the air flow (m³/(m²h)) - positive values indicate exfiltration, negative are for infiltration) for the four facades of the BESTEST building (edition made of solid aerated concrete) exposed to the Danish climate. Const20 (red) faces south, const 31 (blue) faces east, const 40 (pink) faces west, and const 49 (dark grey) faces west.

Figure 6  Monthly average wind direction in the Danish DRY, North = 0º, east = 90º.

Figure 7 shows the monthly annual moisture content in the exterior part of the walls when there is no air flow through the walls. It is seen how the moisture content is generally higher in winter than in summer, and that the north wall has the highest moisture content, and the south wall the lowest. The east and west walls exhibit almost the same moisture content and annual variation, and their level lies between the values for the north and south walls.
Figure 7  Moisture content (kg/kg) calculated without filtration in the wall 1.5 cm behind its exterior surface for each of the four orientations: Const20 (red) south, const 31 (blue) east, const 40 (pink) west, and const 49 (dark grey) west.

With air flow, the similar graph for moisture content in the exterior wall layers can be seen in Figure 8. The general pattern is the same: The north wall has the highest moisture content, and the south wall has the lowest by the end of the year. However, it can now be seen how the east and west walls differ from one another, such the east wall is the driest of the two in the first months of the year, and the west is the driest in the fall. It seems, once again, that weather/wall combinations where infiltration dominates are the incidents that cause the lowest moisture content in the walls.
FURTHER WORK
- Further checking and validation of the model...
- Implementation of Darcy flow model for walls instead/as a supplement to orifice flow model
- Adjustment of indoor air infiltration
- Energy impacts should be analysed, e.g. the dynamic wall principle

CONCLUSION
A preliminary model for filtration air flow through wall has been implemented to investigate its effect on heat and moisture conditions in the wall.

LITERATURE


